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File   2:INSPEC 1969-2005/Jul W3
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         (c) 2005 The HW Wilson Co.
File 144:Pascal 1973-2005/Jul W3
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File 211:Gale Group Newsearch(TM) 2005/Jul 27
         (c) 2005 The Gale Group
File 239:Mathsci 1940-2005/Sep
         (c) 2005 American Mathematical Society
File 275:Gale Group Computer DB(TM) 1983-2005/Jul 27
         (c) 2005 The Gale Group
File 324:German Patents Fulltext 1967-200529
         (c) 2005 Univentio
File 331:Derwent WPI First View    UD=200547
         (c) 2005  Thomson Derwent
File 340:CLAIMS(R)/US Patent 1950-05/Jul 21
         (c) 2005 IFI/CLAIMS(R)
File 342:Derwent Patents Citation Indx 1978-05/200546
         (c) 2005 Thomson Derwent
File 345:Inpadoc/Fam.& Legal Stat 1968-2005/UD=200529
         (c) 2005 EPO
File 348:EUROPEAN PATENTS 1978-2005/Jul W03
         (c) 2005 European Patent Office
File 351:Derwent WPI 1963-2005/UD,UM &UP=200547
         (c) 2005  Thomson Derwent
File 440:Current Contents Search(R) 1990-2005/Jul 27
         (c) 2005 Inst for Sci Info
File 484:Periodical Abs Plustext 1986-2005/Jul W4
         (c) 2005 ProQuest
File 654:US Pat.Full. 1976-2005/Jul 14
         (c) Format only 2005 The Dialog Corp.

Set     Items   Description
S1         51   (AU=ENENKEL, R? OR AU=ENENKEL R?)
S2         34   RD (unique items)
S3          7   S2 AND POLYNOM?
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3/3,K/1 (Item 1 from file: 340)
DIALOG(R)File 340:CLAIMS(R)/US Patent

10789063 2005-0027772
E/INCREASED PRECISION IN THE COMPUTATION OF A RECIPROCAL SQUARE ROOT
Inventors: **Enenkel Robert F** (CA); Goldiez Robert L (US); Ward T J Christopher (US
Assignee: International Business Machines Corp
Assignee Code: 42640

	Publication Number	Kind	Date	Application Number	Date
	US 20050027772	A1	20050203	US 2003632362	20030731
Priority Applic:				US 2003632362	20030731

Inventors: **Enenkel Robert F** ...

Exemplary Claim: ...root of a number; rounding said estimate to a lower precision; computing the residual of said rounded estimate; using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and multiplying said rounded estimate by said residual error and adding the result to...
Non-exemplary Claims: ...estimate to a lower precision; an arrangement for computing the residual of said rounded estimate; an arrangement for using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and an arrangement for multiplying said rounded estimate by said residual error and adding...

...root of a number; rounding said estimate to a lower precision; computing the residual of said rounded estimate; using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and multiplying said rounded estimate by said residual error and adding the result to...

3/3,K/2 (Item 2 from file: 340)
DIALOG(R)File 340:CLAIMS(R)/US Patent

10118688 2002-0062295
E/METHOD AND APPARATUS FOR EVALUATING POLYNOMIALS **AND RATIONAL FUNCTIONS**
Inventors: **Enenkel Robert F** (CA); Keras Sigitas (CA
Assignee: International Business Machines Corp CA

	Publication Number	Kind	Date	Application Number	Date
	US 20020062295	A1	20020523	US 20018473	20011109
Priority Applic:				CA 2325615	20001011

METHOD AND APPARATUS FOR EVALUATING POLYNOMIALS **AND RATIONAL FUNCTIONS**
Inventors: **Enenkel Robert F** ...

Abstract: Disclosed herein are a computer-processing method and apparatus for computing values of **polynomials** or rational functions. A mathematical software library can advantageously embody the concepts of this invention. The method can be adapted to compute values for non-elementary,special functions, for example ERF, ERFC, LGAMMA, and Bessel functions. The steps

for **polynomial** evaluation include presenting input data that includes coefficients of **polynomial** p(x), x, a predetermined xi, and p(xi), building **polynomial** c(x) having coefficients so that **polynomial** p(x) is expressible as: p(x)=p(xi)+(x-xi) middle-dot c(x), determining each coefficient of **polynomial** c(x), determining a value of **polynomial** c(x), and constructing a value of **polynomial** p(x) by determining: p(x)=p(xi)+(x-xi) middle-dot c(x). The method can be adapted for providing a value for a rational function r(x)=p(x)/q(x), which is a ratio of a numerator **polynomial** p(x) and a denominator **polynomial** q(x).

Exemplary Claim: ...physical system, the steps comprising: a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing said property, said **polynomial** p(x) being expressed asp(x)= Sigma (Pj middle-dot xj) where j=0 to n, a value of a quantity x, a value of...
...a value of a predetermined p(xi) correspondingly paired with said predetermined xi; b) building, via said machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, said second **polynomial** c(x) being expressible as:C(x)= Sigma (Ck middle-dot xk) where k=0 to (n-1) so that said first **polynomial** p(x) is expressible as:p(x)=p(xi)+(x-xi) middle-dot c(x), comprising the steps of: i) determining, via said machine processing unit, a value for each ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second **polynomial** c(x) from:Ck= Sigma (P(k+1+j) middle-dot xij) where j=0 to (n-1-k) ;ii) determining, via said machine processing unit, a value of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:C(x)= Sigma (Ck middle-dot xk) where k=0 to (n-1);c) constructing, via said machine processing unit, a value of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:p(x)=p(xi)+(x-xi) middle-dot c(x); and d) outputting, via said machine-processing unit, said value of the first **polynomial** p(x) representing said property of the mathematically modelled physical system.
Non-exemplary Claims: ...and xi is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first **polynomial** p(x...

...4. The machine-implementable method of claim 3 wherein said ordered coefficients of said second **polynomial** c(x) are computed from a mathematical expression derivable from:Ck= Sigma (P(k+1+j) middle-dot xij) where j=0 to (n-1...

...mathematical recurrence expression by comprising further steps for: e) equating, via said machine-processing unit, a value of a highest ordered coefficient of said second **polynomial** c(x) to a value of an identified highest ordered coefficient of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:Cn-1=Pn; and f) computing, via a machine processing unit, a value for each lower ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:Ck-1=xi middle-dot Ck+Pk where k=(n-1) to...

...11. The machine-implementable method of claim 10 wherein said step of determining a value of said second **polynomial** c(x) is computed by using Homer's Rule...

...12. The machine-implementable method of claim 11 for determining a value of a denominator **polynomial** q(x) having identified ordered denominator

coefficients, said denominator **polynomial** q(x) being expressible as:q(x)= Sigma (Qj middle-dot xj) where j=0 to m,comprising further steps of: g) adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator **polynomial** q(x), a value of a predetermined q(xi) correspondingly paired with said predetermined xi; and h) determining, via said machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, said another **polynomial** d(x) being expressible as:d(x)= Sigma (Dk middle-dot xk) where k=0 to (m-1) so that said denominator **polynomial** q(x) is expressible as:q(x)=q(xi)+(x-xi) middle-dot d(x), and a value for the said denominator **polynomial** is resolved...

...13. The machine-implementable method of claim 12 wherein the first **polynomial** p(x) is a numerator **polynomial** p(x), and p(x)-p(xi) is computed, and p(xi) is not read...

...The machine-implementable method of claim 13 for determining a value of a rational function r(x) being expressible as a quotient of said numerator **polynomial** p(x) and said denominator **polynomial** q(x) expressed as...physical system, the steps comprising: a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing said property, said **polynomial** p(x) being expressed asp(x)= Sigma (Pj middle-dot xj) where j=0 to n, a value of a quantity x, a value of...

...a value of a predetermined p(xi) correspondingly paired with said predetermined xi; b) building, via said machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, said second **polynomial** c(x) being expressible as:c(x)= Sigma (Ck middle-dot xk) where k=0 to (n-1) so that said first **polynomial** p(x) is expressible as:p(x)=p(xi)+(x-xi) middle-dot c(x), comprising the steps of: i) determining, via said machine processing unit, a value for each ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second **polynomial** c(x) from:Ck= Sigma (P(k+1+j) middle-dot xij) where j=0 to (n-1-k) ;ii) determining, via said machine processing unit, a value of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:c(x)= Sigma (Ck middle-dot xk) where k=0 to (n-1);c) constructing, via said machine processing unit, a value of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:p(x)=p(xi)+(x-xi) middle-dot c(x); and d) outputting, via said machine-processing unit, said value of the first **polynomial** p(x) representing said property of the mathematically modelled physical system...

...and xi is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first **polynomial** p(x...

...26. The machine of claim 25 wherein said ordered coefficients of said second **polynomial** c(x) are computed from a mathematical expression derivable from:Ck= Sigma (P(k+1+j) middle-dot xij) where j=0 to (n-1...

...mathematical recurrence expression by further comprising: e) means for equating, via said machine processing unit, a value of a highest ordered coefficient of said second **polynomial** c(x) to a value of an identified highest ordered coefficient of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to

determine:Cn-1=Pn; and f) means for computing, via said machine processing unit, a value for each lower ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:Ck+1=xi middle-dot Ck+Pk where k=(n-1) to... 33. The machine of claim 32 wherein the determining means for determining a value of said second **polynomial** c(x) is computed by using Homer's Rule...

...34. The machine of claim 33 for determining a value of a denominator **polynomial** q(x) having identified ordered denominator coefficients, said denominator **polynomial** q(x) being expressible as:q(x)= Sigma (Qj middle-dot xj) where j=0 to m,comprising further steps of: g) adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator **polynomial** q(x), and a value of a predetermined q(xi) correspondingly paired with said predetermined xi; and h) determining, via said machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, said another **polynomial** d(x) being expressible as:d(x)= Sigma (Dk middle-dot xk) where k=0 to (m-1) so that said denominator **polynomial** q(x) is expressible as:q(x)=q(xi)+(x-xi) middle-dot d(x), and a value for the said denominator **polynomial** is resolved...

...35. The machine of claim 34 wherein the first **polynomial** p(x) is a numerator **polynomial** p(x), and p(x)-p(xi) is computed, and p(xi) is not read...

...36. The machine of claim 35 for determining a value of a rational function r(x) being expressible as a quotient of said numerator **polynomial** p(x) and said denominator **polynomial** q(x) expressed as...

3/3,K/3 (Item 1 from file: 345)
DIALOG(R)File 345:Inpadoc/Fam.& Legal Stat

17830916
Basic Patent (No,Kind,Date): CA 2325615 AA 20020510 <No. of Patents: 002>
METHOD AND APPARATUS FOR EVALUATING POLYNOMIALS AND RATIONALFUNCTIONS
METHODE ET APPAREIL D'EVALUATION DE FONCTIONS POLYNOMIALES ET RATIONNELLES (English; French)
Patent Assignee: IBM CANADA (CA)
Author (Inventor): ENENKEL ROBERT F (CA); KERAS SIGITAS (CA)
IPC: *G06F-017/10;
Language of Document: English
Patent Family:

Patent No	Kind	Date	Applic No	Kind	Date	
CA 2325615	AA	20020510	CA 2325615	A	20001110	(BASIC)
US 20020062295	AA	20020523	US 8473	A	20011109	

Priority Data (No,Kind,Date):
CA 2325615 A 20001110

Dialog File: Inpadoc/Fam.& Legal Stat_1968-2005/UD=200529

3/3,K/4 (Item 1 from file: 351)
DIALOG(R)File 351:Derwent WPI

016827227 **Image available**
WPI Acc No: 2005-151509/200516

XRPX Acc No: N05-127833
Number`s reciprocal square root calculating method for microprocessor, involves multiplying rounded piecewise-linear estimate of reciprocal square root of number with residual error and adding result to estimate
Patent Assignee: IBM CORP (IBMC)
Inventor: **ENENKEL R F** ; GOLDIEZ R L; WARD T J C
Number of Countries: 001 Number of Patents: 001
Patent Family:

Patent No	Kind	Date	Applicat No	Kind	Date	Week	
US 20050027772	A1	20050203	US 2003632362	A	20030731	200516	B

Priority Applications (No Type Date): US 2003632362 A 20030731
Patent Details:

Patent No	Kind	Lan	Pg	Main IPC	Filing Notes
US 20050027772	A1		15	G06F-007/38	

Inventor: **ENENKEL R F** ...

Abstract (Basic):
... for a reciprocal square root of a number and rounding the estimate to a lower precision. Residual of the rounded estimate is computed, and a **polynomial** in the residual of the estimate is computed using a Taylor expansion to obtain the residual error. The rounded estimate is multiplied by the residual...

3/3,K/5 (Item 2 from file: 351)
DIALOG(R)File 351:Derwent WPI

014697832 **Image available**
WPI Acc No: 2002-518536/200255
XRPX Acc No: N02-410422
Machine-implementable method for computing property of mathematically modeled physical system, involves determining ordered coefficients of polynomial **representing property using multiple machine processor signals**
Patent Assignee: IBM CANADA LTD (IBMC); INT BUSINESS MACHINES CORP (IBMC)
Inventor: **ENENKEL R F** ; KERAS S
Number of Countries: 002 Number of Patents: 002
Patent Family:

Patent No	Kind	Date	Applicat No	Kind	Date	Week	
US 20020062295	A1	20020523	US 20018473	A	20011109	200255	B
CA 2325615	A1	20020510	CA 2325615	A	20001011	200255	

Priority Applications (No Type Date): CA 2325615 A 20001011
Patent Details:

Patent No	Kind	Lan	Pg	Main IPC	Filing Notes
US 20020062295	A1		15	G06F-015/18	
CA 2325615	A1	E		G06F-017/10	

Machine-implementable method for computing property of mathematically modeled physical system, involves determining ordered coefficients of polynomial **representing property using multiple machine processor signals**
Inventor: **ENENKEL R F** ...

Abstract (Basic):
... An input data including a value of each identified ordered coefficient of a **polynomial** p(x) representing the property. A valve

for each ordered coefficient of another **polynomial** c(x) and the value of the **polynomial** c(x) itself is determined using multiple machine processor signals. A value of the **polynomial** p(x) is constructed using multiple machine processor signals and **polynomial** c(x), according to a specific equation.

... Determined values of **polynomials** or rational functions by approximating the non-elementary special functions. Coefficients of a **polynomial** or rational function associated with each individual table points need not be stored and are computed using simple formulae from one or two stored function...

...Title Terms: **POLYNOMIAL** ;

3/3,K/6 (Item 1 from file: 654)
DIALOG(R)File 654:US Pat.Full.

5977629 **IMAGE Available
Derwent Accession: 2005-151509
UTILITY
Increased precision in the computation of a reciprocal square root
Inventor: **Enenkel, Robert F.** , Markham, CA
Goldiez, Robert L., Apex, NC, US
Ward, T.J. Christopher, Briarcliff Manor, NY, US
Assignee: IBM Corporation, (03), Armonk, NY, US
Correspondence Address: FERENCE & ASSOCIATES, 400 BROAD STREET, PITTSBURGH, PA, 15143, US

	Publication Number	Kind	Date	Application Number	Filing Date
Main Patent	US 20050027772	A1	20050203	US 2003632362	20030731

Fulltext Word Count: 3322

Inventor: **Enenkel, Robert F** ...

Summary of the Invention:

...root of a number; rounding said estimate to a lower precision; computing the residual of said rounded estimate; using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and multiplying said rounded estimate by said residual error and adding the result to...

...estimate to a lower precision; an arrangement for computing the residual of said rounded estimate; an arrangement for using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and an arrangement for multiplying said rounded estimate by said residual error and adding...

...root of a number; rounding said estimate to a lower precision; computing the residual of said rounded estimate; using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and multiplying said rounded estimate by said residual error and adding the result to...

Description of the Invention:

...epr), `e' will be small (less than 2**-13 in the BG/L implementation), so the first four (4) or so terms of the asymptotic

polynomial expansion for `epr' will be sufficient to achieve the desired precision...

...the result is then multiplied by the argument `x' and 1.0 is subtracted from the product to obtain the residual error. In S350, the **polynomial** in the residual error is computed by using a Taylor Expansion where the argument value is the residual error calculated in S340. In S360 the...

...calculation is as follows: 0.3200X0.3200=0.1024, 0.1024X9.000-1.000=-0.07840. At S430, a Taylor Expansion is performed and the **polynomial** in the residual of -0.07840 is calculated to the desired number of terms as follows, using the **polynomial** equation f(x)=x*(-1/2+x*(-{fraction (5/16)}+x*{fraction (35/128)}))) where x=-0.07840, f (-0.07840)=0.04167. At S440...

...estimate to a lower precision; an arrangement for computing the residual of said rounded estimate; an arrangement for using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and an arrangement for multiplying said rounded estimate by said residual error and adding...

Exemplary or Independent Claim(s):
...root of a number;
rounding said estimate to a lower precision;
computing the residual of said rounded estimate;
using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and
multiplying said rounded estimate by said residual error and adding the result to...

...estimate to a lower precision;
an arrangement for computing the residual of said rounded estimate;
an arrangement for using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and
an arrangement for multiplying said rounded estimate by said residual error and adding...

...root of a number;
rounding said estimate to a lower precision;
computing the residual of said rounded estimate;
using a Taylor Expansion to compute the **polynomial** in said residual of said estimate to obtain the residual error; and
multiplying said rounded estimate by said residual error and adding the result to...

3/3,K/7 (Item 2 from file: 654)
DIALOG(R)File 654:US Pat.Full.

0005007633 **IMAGE Available
Derwent Accession: 2002-518536
Method and apparatus for evaluating polynomials **and rational functions**
Inventor: **Robert Enenkel** , INV
Sigitas Keras, INV
Assignee: International Business Machines Corporation(03), Armonk, NY
Correspondence Address: A. Bruce Clay IBM Corporation T81/503, PO Box 12195 , Research Triangle Park, NC, 27709, US

	Publication Number	Kind	Date	Application Number	Filing Date
Main Patent	US 20020062295	A1	20020523	US 20018473	20011109
Priority				CA 2325615	20001011

Fulltext Word Count: 12761

Method and apparatus for evaluating polynomials **and rational functions**
Inventor: **Robert Enenkel** ...

Abstract:
Disclosed herein are a computer-processing method and apparatus for computing values of **polynomials** or rational functions. A mathematical software library can advantageously embody the concepts of this invention. The method can be adapted to compute values for non-elementary,special functions, for example ERF, ERFC, LGAMMA, and Bessel functions. The steps for **polynomial** evaluation include presenting input data that includes coefficients of **polynomial** p(x), x, a predetermined x[sub]i, and p(x[sub]i), building **polynomial** c(x) having coefficients so that **polynomial** p(x) is expressible as...

...determining each coefficient of **polynomial** c(x), determining a value of **polynomial** c(x), and constructing a value of **polynomial** p(x) by determining...

...which is a ratio of a numerator **polynomial** p(x) and a denominator **polynomial** q(x...

Summary of the Invention:
...for computing values of mathematical expressions, and more specifically to a computer processing method and computer apparatus for computing binary representations of numerical values of **polynomials** and rational functions...

...0002] **Polynomials** and rational functions, which are quotients of **polynomials** , are used, for example, by various branches of applied science for determining numerical values of mathematical expressions for modeling a property of a physical system, for example a rate of air flow over an airfoil surface. **Polynomials** can be used to approximate complicated mathematical expressions. Various methods, for example Homer's Rule that was disclosed in 1819, are used for computing values of **polynomials** , and provide a degree of accuracy that may not be acceptable in certain situations, depending on the particular **polynomial** and the accuracy required. G. E. Forsythe et al in "Computer Methods for Mathematical Computations" Prentice-Hall (1977) discloses, in Section 4.2, using Homer's Rule for computing a numerical value of a **polynomial** .

[...
...modelled physical system, the steps including: reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing the property, the **polynomial** p(x) being expressed as...

...value of a predetermined p(x) correspondingly paired with said predetermined x[sub]i; building, via the machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, the second **polynomial** c(x) being expressible as...

...0010] so that the first **polynomial** p(x) is expressible as...

...0011] including the steps of: determining, via the machine processing unit, a value for each ordered coefficient of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each of the ordered coefficients of the second **polynomial** c(x) from...

...0012] determining, via the machine processing unit, a value of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine...
...0013] and constructing, via the machine processing unit, a value of the first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine...

...0014] and outputting, via the machine-processing unit, the value of the first **polynomial** p(x) representing said property of the mathematically modelled physical system...

...sub]i is sufficiently small to achieve a desired accuracy of a final computation of the machine representation of a numerical value of the first **polynomial** p(x...

...0017] Preferably the machine-implementable method is further adapted so that the ordered coefficients of the second **polynomial** c(x) are computed from a mathematical expression derivable from...

...backward mathematical recurrence expression by including further steps for: equating, via the machine-processing unit, a value of a highest ordered coefficient of the second **polynomial** c(x) to a value of an identified highest ordered coefficient of the first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine...
...0022] and computing, via a machine processing unit, a value for each lower ordered coefficient of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine...

...0025] Preferably the machine-implementable method is further adapted so that the step of determining a value of the second **polynomial** c(x) is computed by using Horner's Rule...

...0026] Preferably the machine-implementable method is further adapted for determining a value of a denominator **polynomial** q(x) having identified ordered denominator coefficients, the denominator **polynomial** q(x) being expressible as...

...0027] including further steps of: adapting the input data to further include a value for each identified ordered denominator coefficient of the denominator **polynomial** q(x), a value of a predetermined q(x) correspondingly paired with the predetermined x[sub]i; and determining, via the machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, the another **polynomial** d(x) being expressible as...

...0028] so that the denominator **polynomial** q(x) is expressible as...
...0029] and a value for the denominator **polynomial** is resolved...

...0030] Preferably the machine-implementable method is further adapted so that the first **polynomialp** (x) is a numerator **polynomialp** (x), and p(x)-p(x[sub]i) is computed, and p(x[sub]i) is not read...

...the machine-implementable method is further adapted for determining a value of a rational function r(x) being expressible as a quotient of the numerator **polynomial** p(x) and the denominator **polynomial** q(x) expressed as...

...modelled physical system, the steps including: reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing the property, the **polynomial** p(x) being expressed as...

...a predetermined p(x[sub]i) correspondingly paired with the predetermined x[sub]i; building, via the machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, the second **polynomial** c(x) being expressible as...

...0043] so that the first **polynomial** p(x) is expressible as...
...0044] including the steps of: determining, via the machine processing unit, a value for each ordered coefficient of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each of the ordered coefficients of the second **polynomial** c(x) from...

...0045] determining, via the machine processing unit, a value of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine...

...0046] constructing, via the machine processing unit, a value of the first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine...

...0047] and outputting, via the machine-processing unit, the value of the first **polynomial** p(x) representing the property of the mathematically modelled physical system...

...sub]i is sufficiently small to achieve a desired accuracy of a final computation of the machine representation of a numerical value of the first **polynomial** p(x...
...0050] Preferably, the machine is further adapted so that the ordered coefficients of the second **polynomial** c(x) are computed from a mathematical expression derivable from...

...backward mathematical recurrence expression by further including: means for equating, via the machine processing unit, a value of a highest ordered coefficient of the second **polynomial** c(x) to a value of an identified highest ordered coefficient of the first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine...

...0055] and means for computing, via the machine processing unit, a value for each lower ordered coefficient of the second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine...

...0058] Preferably, the machine is further adapted for determining means for determining a value of the second **polynomial** c(x) is computed by using Homer's Rule...

...0059] Preferably, the machine is further adapted for determining a value of a denominator **polynomial** q(x) having identified ordered denominator coefficients, the denominator **polynomial** q(x) being expressible as...

...0060] including further steps of: adapting the input data to further include a value for each identified ordered denominator coefficient of the denominator **polynomial** q(x), and a value of a predetermined q(x[sub]i) correspondingly paired with the predetermined x[sub]i; and determining, via the machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, the another **polynomial** d(x) being expressible as...

...0061] so that the denominator **polynomial** q(x) is expressible as...

...0062] and a value for the denominator **polynomial** is resolved...

...0063] Preferably, the machine is further adapted so that the first **polynomial** p(x) is a numerator **polynomial** p(x), and p(x)-p(x[sub]i) is computed, and p(x[sub]i) is not read...

...0064] Preferably, the machine is further adapted for determining a value of a rational function r(x) being expressible as a quotient of the numerator **polynomial** p(x) and the denominator **polynomial** q(x) expressed as

Description of the Drawings:
...0075]FIG. 1 depicts a programming flow chart for computing a value of a **polynomial** p(x) in accordance with the present invention...

Description of the Invention:
...described with reference to an exemplary context of a computer processing method and apparatus for computing a binary representation of a numerical value of a **polynomial** p(x...

...0080] A **polynomial** p(x) can be mathematically expressed as shown in the following equation...

...0081] Ordered coefficients of **polynomial** p(x) are P[sub]0, . . . , P[sub]n. **Polynomial** p(x) has n+1 ordered coefficients and is said to have degree n. A numerical value of **polynomial** p(x) can be computed by reading each coefficient of **polynomial** p(x) and an identified x and using steps in Equation 1. Following Equation 1 directly would require a computer processor to perform...

...0082] multiplications and n additions. To significantly reduce the processing time, Homer's Rule can be applied to Equation 1, in which **polynomial** p(x) can be expressed in Equation 1a, as shown below...

...0083] By following Equation 1a, a program can perform n multiplications and n additions to compute a number for **polynomial** p(x) using less processing time than a program that follows Equation 1...

...0084] A **polynomial** p(x) at x=x[sub]i can be expressed as follows...

...first sum k=0 to (n-1), and in the second sum j=0 to (n-1-k) By following Equation 4a, a value for **polynomial** p(x) can be computed by reading coefficients of the **polynomial** (P[sub]1, P[sub]2, . . . , P[sub]n), and x, x[sub]i, and p(x[sub]i). Equation 4a can be expressed as ...

...0089] Equation 5 includes an expression for a new **polynomial** c(x) having coefficients, from C[sub]0, . . . , C[sub](n-1), that are not known a priori and therefore need to be determined. Equation 6 can be used for determining values of the coefficients of new **polynomial** c(x)

by involving the coefficients of **polynomial** p(x) and an identified x[sub]i. Optionally, other expressions for determining values for each coefficient of new **polynomial** c(x) can be derived from Equation 6...

...0090] It is appreciated that Homer's Rule could be used to compute a value for **polynomial** c(x) after coefficients of **polynomial** c(x) have been computed by using Equation 6 or optionally from another expression that could be derived from Equation 6. By using Homer's...
...0091] Processing steps that follow Equation 5a begin within the innermost set of parentheses and proceed outwards to the outermost brackets. Calculated coefficients of new **polynomial** c(x) can subsequently be used in Equation 5a for computing a numerical value of **polynomialp** (x...

...0092] Optionally, unknown coefficients of new **polynomial** c(x) can be computed by applying the principle of mathematical recurrence on Equation 6. For example, a forward mathematical recurrence can allow computation of coefficients of **polynomial** c(x) by beginning with the lowest ordered coefficient, C[sub]0, and proceeding to the highest ordered coefficient, C[sub](n-1). Optionally, a backward mathematical recurrence can compute coefficients of **polynomial** c(x) by beginning with the highest ordered coefficient, C[sub](n-1) and proceeding to the lowest ordered coefficient, C[sub]0. With each step of backwardly computing a coefficient of **polynomial** c(x), a next outer most bracketed term of Equation 5a can be advantageously computed...

...0093] Optionally, a backward recurrence for determining each coefficient of new **polynomial** c(x) can be realized by using Equation 7 and Equation 8. Each subsequent lower-ordered unknown coefficient of new **polynomial** c(x) can be computed by using Equation 8...
...0094] To compute a numerical value of a **polynomial** p(x), a computer can read input data that includes a value for each identified ordered coefficient of the first **polynomial** p(x), a value of a quantity x, a value of a predetermined x[sub]i, a value of a predetermined p(x[sub]i...

...a set of predetermined x[sub]i, and a nearest predetermined p(x[sub]i) corresponds to the nearest x[sub]i. Optionally, coefficients of **polynomial** c(x) can be computed by following Equation 7 and Equation 8. A value for **polynomial** p(x) can then be computed by following Equation 5a. The operations of computing the coefficients of **polynomial** c(x) in Equation 8 can be interleaved with computing the nested parenthesis in Equation 5a so that each time a new C[sub]k...

...computational purpose, a difference between x and x[sub]i can be sufficiently small enough to achieve a desired accuracy of a computed value of **polynomial** p(x...

...which is represented as a computer processing method embodied in a programming flow chart for computing a machine representation of a numerical value of a **polynomial** p(x). Optionally, coefficients of p(x) can be placed in a vector of length n. The method optionally includes a step of accessing a...
...0098] In step 10, the method of the preferred embodiment begins. In step 15, each coefficient of **polynomial** p(x) and an identified x can be read by a computer processing unit. In step 20, the computer optionally selects a predetermined x[sub...

...Variable ksum is an ongoing summation that represents the sum in Equation 5a. Variable ksum is initially set equal to coefficient P[sub]n of **polynomial** p(x). Variable ck is a temporary value for any

coefficients of **polynomial** c(x), in which variable ck is computed to be a specific coefficient of **polynomial** c(x) that is determined in step 50 as will be shown below. Variable ck is initially set equal to P[sub]n, which is the highest ordered coefficient of **polynomial** p(x). Variable k is an index counter for a processing loop, in which variable k is initially set equal to (n-1), which is one less than the degree of **polynomial** p(x). In step 40, a query is performed to determine whether a value for variable k is equal to zero. If the numerical value...

...variable ck is computed, which is coefficient C[sub]k by using the backward recurrence of Equation 8 that involves a higher ordered coefficient of **polynomial** p(x), which is P[sub]k, and a previously determined coefficient, C[sub]k-1, of **polynomial** c(x). Next, an updated value for variable ksum is computed, which is an ongoing summation of the sum in Equation 5a, by determining a...

...is computed so that the sum in Equation 5a can be resolved towards its outermost brackets. In step 60, the method uses a value for **polynomial** c(x), which is expressed in FIG. 1 as variable ksum after variable k equals zero in step 40. A value of **polynomial** p(x) is computed as a rearrangement of Equation 5a, which is expressed as...

...machine representation of a numerical value of a rational function r(x). The rational function r(x) is expressed as a quotient of a numerator **polynomial** p(x) having a set of n predetermined coefficients from P[sub]0, P[sub]1, . . . , P[sub]n, so that...

...0101] and a denominator **polynomial** q(x) having m predetermined coefficients from Q[sub]0, Q[sub]1, . . . , Q[sub]m, so that...

...0107] The second method computes values of a numerator **polynomial** p(x) and a denominator **polynomial** q(x) by following Equation 5, as follows...

...0108] **Polynomial** c(x) has (n-1) unknown coefficients and **polynomial** d(x) has (m-1) unknown coefficients. The second method can determine coefficients of c(x) and d(x) and then determine a value for...

...0110] In step 110, the second method begins. In step 115, a computer reads values of each coefficient of **polynomial** p(x) and each coefficient of **polynomial** q(x), and an identified x. In step 120, a predetermined x[sub]i from a set of predetermined values of x[sub]i can ...

...value for variable dp (i.e., delta p) is computed,which is a computed value for p(x)-p(x[sub]i) for a numerator **polynomialp** (x). Variable dp will be used in step 205 as will be shown later. Step 170, step 180, and step 190 are similar to step...

...step 200 a value for variable dq (i.e., delta q), which is a computed value for q(x)-(x[sub]i) for a denominator **polynomial** q(x) is computed. Variable dq will be used in step 202 and 205 as shown later. In step 202 a numerical value for q...

...by first approximating the Bessel function by a piece-wise rational function. Bessel functions can typically be approximated by rational functions having numerator and denominator **polynomials** with the same degree (i.e. same number of coefficients...

...value of a rational function is desired to be computed, and P and Q, which are vectors containing the coefficients of the numerator and

denominator **polynomials** p(x) and q(x), respectively, of the rational function, and T, which is a table containing a set of predetermined x[sub]i's...

...n, which is the degree of p(x) and q(x). In step 362 a computer processing unit reads values for each coefficient of numerator **polynomial** p(x) and denominator **polynomial** q(x), in which both **polynomials** have the same number or coefficients, and also reads an identified x. In step 364, to achieve improved performance, a nearest x[sub]i and...

...In step 366 various variables are initialized for subsequent use in an instruction loop. One software loop is used because the numerator and the denominator **polynomials** have the same degree (that is, share the same number of coefficients). This loop is similar to a combination of the two loops involving steps...

...the elementary functions, for these non-elementary special functions, would require a computer processing unit to read coefficients of each of a large number of **polynomials** , one associated with each table point. This can make the table unacceptably large...

...0117] Computing steps that embody the methods of the present invention can be used to determine values for **polynomials** or rational functions approximating the non-elementary special function. Optionally, coefficients of a **polynomial** or rational function associated with each individual table point, x[sub]i, do not have to be stored; instead, these coefficients can be computed using simple formulas from only one (for **polynomials**) or two (for rational functions) stored function values per table point, resulting in tables of manageable size. In this sense, the present invention provides a...

...a physical system. C. M. Close in "The Analysis of Linear Circuits", Harcourt, Brace & World (1966), discloses in Chapters 6 and 11 the use of **polynomials** for determining frequency response characteristics of electrical systems. The same concepts can be applied to non-electrical systems, such as mechanical, hydraulic, acoustical, and thermal systems. R. C. Weyrick in "Fundamentals of Automatic Control", McGraw-Hill Book Company (1975), discloses in Chapters 2 and 6 the use of **polynomials** for mathematically modelling physical systems for predicting a behavior of a physical system. D. Halliday and R. Resnick in "Physics: Parts 1 and 2", John Wiley & Sons (1978) disclose in Chapters 3 and 4 the use of **polynomials** for mathematically modelling and predicting spatial co-ordinates of impact of a projectile being released from an aeroplane ...

Exemplary or Independent Claim(s):

...physical system, the steps comprising: a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing said property, said **polynomial** p(x) being expressed asp(x)=[capital Sigma, Greek](P[sub]j[middle dot]x[sup]j) where j=0 to n,a value of ...

...predetermined p(x[sub]i) correspondingly paired with said predetermined x[sub]i; b) building, via said machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, said second **polynomial** c(x) being expressible as:C(x)=[capital Sigma, Greek](C[sub]k[middle dot]x[sup]k) where k=0 to (n-1)so that said first **polynomial** p(x) is expressible

as:p(x)=p(x[sub]i)+{x-x[sub]i}[middle dot]c(x),comprising the steps of: i) determining, via said machine processing unit, a value for each ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second **polynomial** c(x) from:C[sub]k=[capital Sigma, Greek](P[sub](k+1+j)[middle dot]x[sub]i[sup]j) where j=0 to (n-1-k);ii) determining, via said machine processing unit, a value of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:C(x)=[capital Sigma, Greek](C[sub]k[middle dot]x[sup]k) where k=0 to (n-1);c) constructing, via said machine processing unit, a value of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:p(x)=p(x[sub]i)+{x-x[sub]i}[middle dot]c(x);and d) outputting, via said machine-processing unit, said value of the first **polynomial** p(x) representing said property of the mathematically modelled physical system...

...physical system, the steps comprising: a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first **polynomial** p(x) representing said property, said **polynomial** p(x) being expressed asp(x)=[capital Sigma, Greek](P[sub]j[middle dot]x[sup]j) where j=0 to n,a value of ...

...predetermined p(x[sub]i) correspondingly paired with said predetermined x[sub]i; b) building, via said machine processing unit, a value of a second **polynomial** c(x) having ordered coefficients, said second **polynomial** c(x) being expressible as:c(x)=[capital Sigma, Greek](C[sub]k[middle dot]x[sup]k) where k=0 to (n-1)so that said first **polynomial** p(x) is expressible as:p(x)=p(x[sub]i)+{x-x[sub]i}[middle dot]c(x),comprising the steps of: i) determining, via said machine processing unit, a value for each ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second **polynomial** c(x) from:C[sub]k=[capital Sigma, Greek](P[sub](k+1+j)[middle dot]x[sub]i[sup]j) where j=0 to (n-1-k);ii) determining, via said machine processing unit, a value of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:c(x)=[capital Sigma, Greek](C[sub]k[middle dot]x[sup]k) where k=0 to (n-1);c) constructing, via said machine processing unit, a value of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:p(x)=p(x[sub]i)+{x-x[sub]i}[middle dot]c(x);and d) outputting, via said machine-processing unit, said value of the first **polynomial** p(x) representing said property of the mathematically modelled physical system.

Non-exemplary or Dependent Claim(s):

...sub]i is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first **polynomial** p(x...

...4. The machine-implementable method of claim 3 wherein said ordered coefficients of said second **polynomial** c(x) are computed from a mathematical expression derivable from:C[sub]k=[capital Sigma, Greek](P[sub](k+1+j)[middle dot]x[sub...

...mathematical recurrence expression by comprising further steps for: e)

equating, via said machine-processing unit, a value of a highest ordered coefficient of said second **polynomial** c(x) to a value of an identified highest ordered coefficient of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:C[sub]n-1=P[sub]n;and f) computing, via a machine processing unit, a value for each lower ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:C[sub]k-1=x[sub]i[middle dot]C[sub]k...

...11. The machine-implementable method of claim 10 wherein said step of determining a value of said second **polynomial** c(x) is computed by using Homer's Rule...

...12. The machine-implementable method of claim 11 for determining a value of a denominator **polynomial** q(x) having identified ordered denominator coefficients, said denominator **polynomial** q(x) being expressible as:q(x)=[capital Sigma, Greek](Q[sub]j[middle dot]x[sup]j) where j=0 to m,comprising further steps of: g) adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator **polynomial** q(x), a value of a predetermined q(x[sub]i) correspondingly paired with said predetermined x[sub]i; and h) determining, via said machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, said another **polynomial** d(x) being expressible as:d(x)=[capital Sigma, Greek](D[sub]k[middle dot]x[sup]k) where k=0 to (m-1)so that said denominator **polynomial** q(x) is expressible as:q(x)=q(x[sub]i)+{x-x[sub]i}[middle dot]d(x),and a value for the said denominator **polynomial** is resolved...

...13. The machine-implementable method of claim 12 wherein the first **polynomial** p(x) is a numerator **polynomial** p(x), and p(x)-p(x[sub]i) is computed, and p(x[sub]i) is not read...

...The machine-implementable method of claim 13 for determining a value of a rational function r(x) being expressible as a quotient of said numerator **polynomial** p(x) and said denominator **polynomial** q(x) expressed as...

...sub]i is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first **polynomial** p(x...

...26. The machine of claim 25 wherein said ordered coefficients of said second **polynomial** c(x) are computed from a mathematical expression derivable from:C[sub]k=[capital Sigma, Greek](P[sub](k+1+j)[middle dot]x[sub...

...mathematical recurrence expression by further comprising: e) means for equating, via said machine processing unit, a value of a highest ordered coefficient of said second **polynomial** c(x) to a value of an identified highest ordered coefficient of said first **polynomial** p(x) by generating a plurality of machine processing unit signals to determine:C[sub]n-1=P[sub]n;and f) means for computing, via said machine processing unit, a value for each lower ordered coefficient of said second **polynomial** c(x) by generating a plurality of machine processing unit signals to determine:C[sub]k+1=x[sub]i[middle dot]C[sub]k...

...33. The machine of claim 32 wherein the determining means for

determining a value of said second **polynomial** c(x) is computed by using Homer's Rule...

...34. The machine of claim 33 for determining a value of a denominator **polynomial** q(x) having identified ordered denominator coefficients, said denominator **polynomial** q(x) being expressible as:q(x)=[capital Sigma, Greek](Q[sub]j[middle dot]x[sub]j) where j=0 to m,comprising further steps of: g). adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator **polynomial** q(x), and a value of a predetermined q(x[sub]i) correspondingly paired with said predetermined x[sub]i; and h) determining, via said machine processing unit, a value of another **polynomial** d(x) having ordered denominator coefficients, said another **polynomial** d(x) being expressible as:d(x)=[capital Sigma, Greek](D[sub]k[middle dot]x[sup]k) where k=0 to (m-1)so that said denominator **polynomial** q(x) is expressible as:q(x)=q(x[sub]i)+{x-x[sub]i}[middle dot]d(x),and a value for the said denominator **polynomial** is resolved...

...35. The machine of claim 34 wherein the first **polynomial** p(x) is a numerator **polynomial** p(x), and p(x)-p(x[sub]i) is computed, and p(x[sub]i) is not read...

...36. The machine of claim 35 for determining a value of a rational function r(x) being expressible as a quotient of said numerator **polynomial** p(x) and said denominator **polynomial** q(x) expressed as